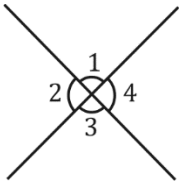
Geometry

1. **Polygon:** Two types of angle
   i. Exterior
   ii. Interior

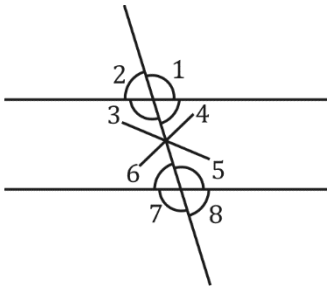
   i. Exterior Angle: Sum of exterior Angle of the polygon is 360°,
   If the polygon is regular polygon then each exterior angle is $\frac{360^\circ}{n}$, where 'n' is number of sides.
   ii. Interior Angle: Sum of the interior angle of the polygon is (n - 2)× 180.
   If the polygon is regular polygon then each interior angle is equal to $\frac{(n-2)180}{n}$
   Number of diagonals of the polygon $= \frac{n\times(n-3)}{2}$

2. Vertical opposite angle always be same

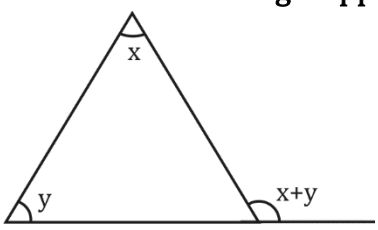


   -: $\angle 1 = \angle 3$ and $\angle 1 + \angle 2 = 180^\circ$
   -: $\angle 2 = \angle 4$ and $\angle 3 + \angle 4 = 180^\circ$

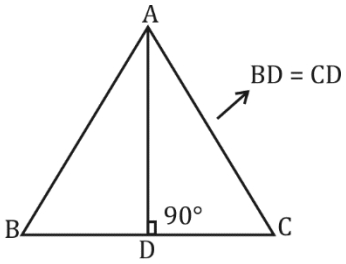
3. **Corresponding angles**:
   $\angle 4 + \angle 5 = 180^\circ$
   $\angle 3 + \angle 6 = 180^\circ$



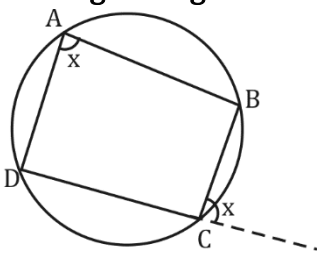
4. **Sum of 2 interior angle opposite to exterior angle**



5. In the given fig. AB = AC, then AD which is median of the triangle also be height of triangle



➢ **In the given fig. ABCD is a cyclic quadrilateral.**



$\angle A + \angle C = 180°$ (opp. Angle)

$\angle B + \angle D = 180°$

$\Longrightarrow$ opposite interior angle is equal to exterior angle.

➢ **Centres of the triangle:**

Type of centres:

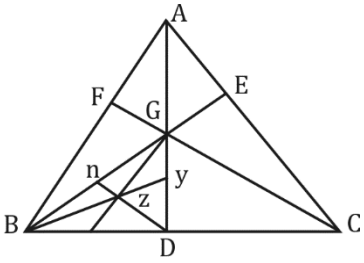
(1) Centroid

(2) Incentre

(3) Circum-centre

(4) Ortho – centre

(1) Centroid: Intersecting points of the medians of triangle is known as centroid of the triangle.



Area of Δ ABD = Δ ACD

AG : GD = 2 : 1

Area of Δ BGC = Δ AGC = Δ AGB

Area of Δ nzGY : Δ ABC

2 : 36

1 : 18

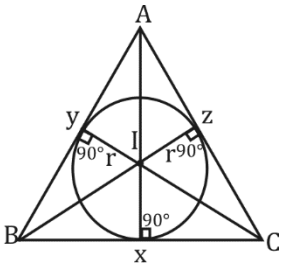
Example: PS is the median of a triangle PQR and O is centroid such that PS = 27 cm. The length of PO is

Sol. PS is the median and O is the centroid ----- (given)

PS = 27 cm

Ratio of PO : OS

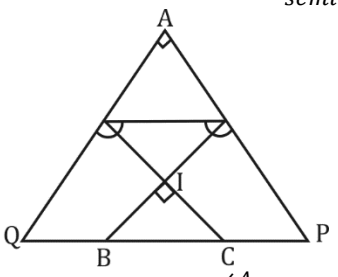
(2) Incentre: Intersecting points of angle bisector of triangle is known as Incentre of the triangle



Ix = Iy = Iz = radius

$\angle BIC = 90 + \frac{\angle A}{2}$

Incircle radius (r) = $\frac{radius\ of\ \Delta}{semi\ perimeter}$

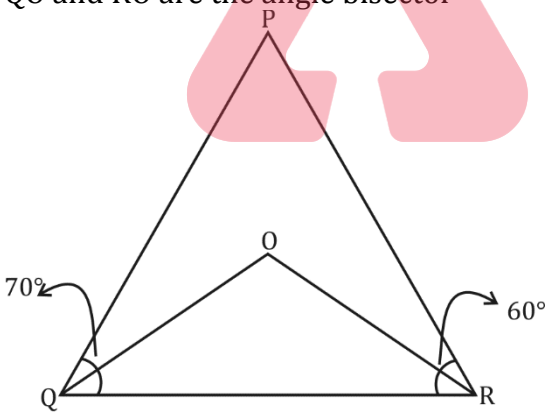


$\Longrightarrow \angle BIC = 90 - \frac{\angle A}{2}$

Example: O is the incentre of triangle PQR, ∠ PQR = 70° and ∠PRQ = 60°, Then find the value of ∠ QOR.

Sol. Acc. to Question

QO and RO are the angle bisector



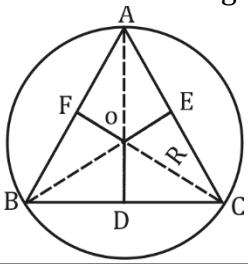
∴ ∠ RQO = 35° and ∠ QRO = 30°

In Δ QOR, ∠ RQO + ∠ QRO + ∠ QOR = 180°

35° + 30° + ∠ QOR = 180°

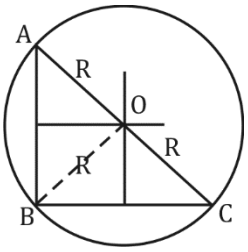
∠ QOR = 180° - 65° ⟹ 115°

(3) Circum-centre: Intersecting point of the perpendicular bisector of triangle is known as circum-centre of the triangle

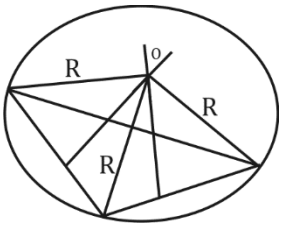


AO = OB = OC = Radius

∠BOC = 2 (∠BAC)

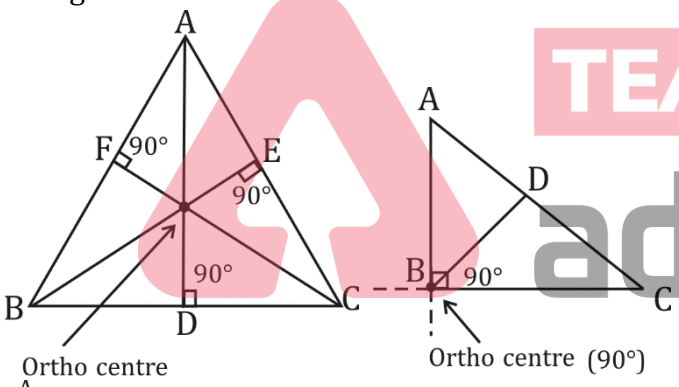


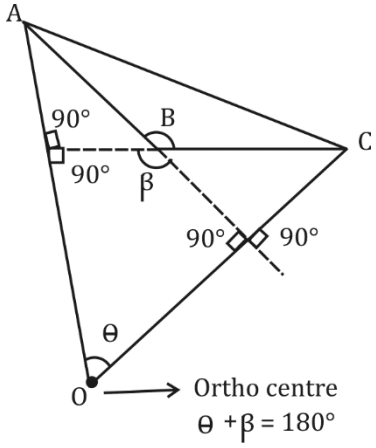
In right ∠ Δ circum-centre is formed on the mid-point of hypotenuse.



circum-circle radius (R)= $\frac{AbC\ (sides)}{4\times area\ of\ \Delta}$

(4) Ortho-centre: intersecting points of the altitudes of triangle is known as orthocentre of the triangle





Example: In an obtuse angled triangle ABC, ∠B is obtuse angled and O is orthocentre. ∠ AOC = 69° and ∠ ABC is

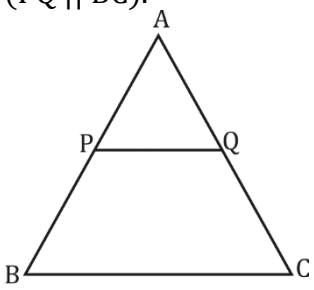
Sol. ∠ ABC = 180° - ∠ AOC

= 180° - 69°

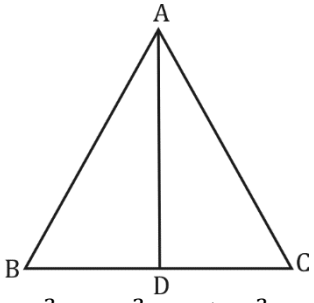
= 111°

Some important facts of the triangle:

i. **Mid-Point Theorem:** In triangle ABC, P and Q are mid – point of AB and AC. Then PQ always || to BC (PQ || BC).



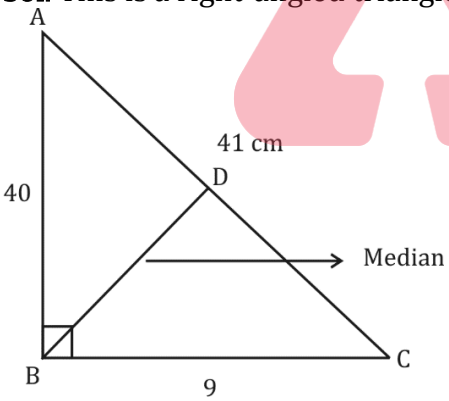
ii. **Median theorem:** In Δ ABC, AD is Median



$AB^2 + AC^2 = 2(AD^2 + BD^2)$

Example: If the length of the three sides of a triangle is a 9 cm, 40 cm, and 41 cm then find the length of median to its greatest side.

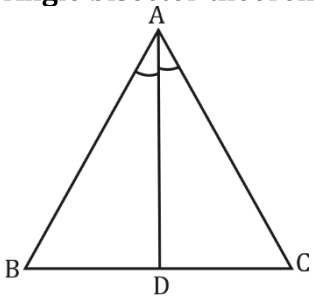
Sol. This is a right-angled triangle



In Right angle triangle medians divide the hypotenuse in 2 equal parts

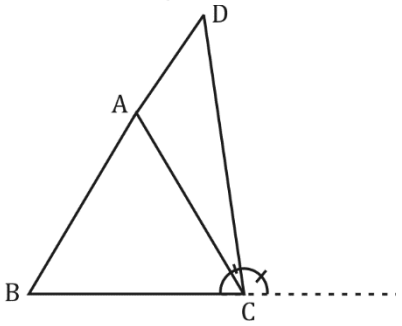
So, $BD = \frac{H}{2} \Longrightarrow \frac{41}{2} = 20.5\ cm$

iii. **Angle bisector theorem:** Internal angle bisector



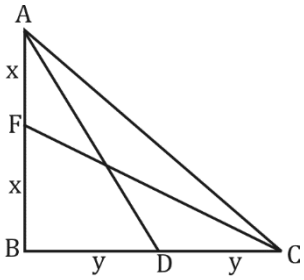
$\Longrightarrow \frac{AB}{AC} = \frac{BD}{CD}$

External angle bisector:



$\frac{BC}{AC} = \frac{BD}{AD}$

- In the right Triangle ABC, F and D is the mid – points of AB and BC



$AC^2 = 4x^2 + 4y^2$

$AD^2 = 4x^2 + y^2$

$FC^2 = 4y^2 + x^2$

$CF^2 + AD^2 = 5(x^2 + y^2)$

$\frac{AD^2}{4} = x^2 + y^2$

$4(CF^2 + AD^2) = 5AD^2$